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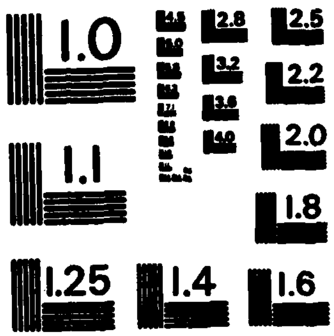

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COLD WAVEGUIDE ELECTROSTATIC MODES BETWEEN UPPER HYBRID AND PLASMA RESONANCE

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PPG-640 July, 1982

Contract N00014-75-C-0476

CENTER FOR
PLASMA PHYSICS
AND
FUSION ENGINEERING

UNIVERSITY OF CALIFORNIA
LOS ANGELES

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Abstract

The structure of electrostatic oscillations is determined for a cold plasma with a zero order density gradient along the magnetic field. A set of well-behaved structures, bounded at plasma resonance, exists for discrete values of perpendicular wavenumber. Under these conditions the plasma acts as a waveguide. A Green's function suitable for sources located in the overdense region of the plasma is constructed from these waveguide-like solutions.

I. Introduction

Magnetized plasmas with a zero order density gradient (∇N_o) along the magnetic field ($\underset{\sim}{B}_o$) can support a rich variety of electrostatic oscillations. The variety of mode structures in this geometry arises from the close juxtaposition of the plasma (ω_p) and upper hybrid (ω_{uh}) resonances for wave frequencies ω greater than the electron gyrofrequency Ω, as sketched in Fig. 1. The majority of theoretical studies have concentrated on two extreme cases. Either the plasma is unmagnetized ($\underset{\sim}{B}_o=0$) and the electrostatic structure associated with the plasma resonance is studied, or the density gradient is perpendicular to the magnetic field ($\nabla N_o \cdot \underset{\sim}{B}_o=0$) and the electrostatic structures around ω_{uh} are investigated. Each of these two cases has received considerable theoretical[1,2] and experimental[1,3] attention and numerous interesting effects have been found for each case.

In the geometry considered here ($\nabla N_o \parallel \underset{\sim}{B}_o$) the effects of both resonances must be considered simultaneously. In fact, WKB analysis of a wave propagating obliquely to $\underset{\sim}{B}_o$ indicates that the upper hybrid resonance is actually a cutoff for parallel wavenumber ($k_\parallel=0$) while the plasma resonance is a true resonance (in that $k_\parallel \to \infty$). Due to the coupling between the ω_p and ω_{uh} points and the need to consider mode conversion near the ω_p point, the general problem of wave excitation by an external source is rather complicated both physically and mathematically. Although we have made some progress towards a solution of the general problem we concentrate our attention here on a rather specific but interesting aspect of the electrostatic mode structures. Specifically, we concentrate on a description of new, purely cold, mode structures which are supported by the plasma for a discrete set of perpendicular wavenumbers satisfying the condition,

$$k_{\perp}\Delta z = [1 - (\Omega/\omega)^2]^{\frac{1}{2}}(2j + 1) \qquad j = 0,1,2,\ldots \tag{1}$$

Where Δz refers to the distance between the upper hybrid and plasma resonance points and Ω is the electron gyrofrequency. These modes are trapped in the region between the ω_p and ω_{uh} points. The wave energy propagates across the magnetic field and the region behaves as a waveguide. We demonstrate that exact solutions to the cold plasma differential equation can be found that are well behaved when the condition given in Eq. (1) is satisfied. With these solutions we construct a Green's function suitable for sources located in the overdense region of the plasma ($\omega < \omega_p$).

Aside from the intrinsic interest of the quantized mode structures, their existence can be of relevance to the understanding of resonant absorption of radio waves in the auroral ionosphere or in certain laboratory devices. In addition this study gives a clear example of a mode structure which depends explicitly upon the plasma inhomogenity and is not contained in the WKB description which typically forms the basis for physical intuition.

II. Mode Structure

Consider an electrostatic oscillation propagating in an inhomogeneous plasma in which the zero order density gradient points along the magnetic field with a waveform given by

$$\Phi = \phi(z) \exp[i(k_\perp x - \omega t)] + c.c. \tag{2}$$

The direction along the gradient is denoted by z while x denotes the direction perpendicular to it, as illustrated in Fig. 1. The spatial structure of $\phi(z)$ is determined by Poisson's equation

$$\partial_z(\varepsilon_\parallel \partial_z \phi) - k_\perp^2 \varepsilon_\perp \phi = S(z) \tag{3}$$

where the perpendicular and parallel components of the dielectric tensor are $\varepsilon_\perp$ and $\varepsilon_\parallel$, respectively. In a cold plasma with varying electron density

$$\varepsilon_\perp = 1 - \omega_p^2(z)/(\omega^2 - \Omega^2) \ ; \quad \varepsilon_\parallel = 1 - \omega_p^2(z)/\omega^2 \tag{4}$$

where $\omega_p^2(z) = 4\pi e^2 N_o(z)/m$ with e,m the electron charge and mass. In Eq. (3), S(z) represents a source of oscillations at frequency ω which may consist, for example, of externally driven grids immersed in the plasma or of charge oscillations induced on the density gradient by a remotely launched electromagnetic wave. Regardless of the particular nature of the source the system response can be found by solving Eq. (3) with an idealized delta function driver. Mathematically, this is equivalent to solving for the Green's function. Once the Green's function has been found the system response to an arbitrary source can be calculated.

The Green's function for Eq. (3) can be constructed from linear combinations of the solutions to the corresponding homogeneous equation (i.e. $S(z)=0$) which satisfy the proper boundary conditions. To obtain solutions to the homogeneous equation we consider a density profile which varies linearly with z in the region of interest. The dielectric components then take the form

$$\varepsilon_{\parallel} = z/L \; ; \quad \varepsilon_{\perp} = (z/L - Y^2)/(1 - Y^2) \tag{5}$$

where $Y = \Omega/\omega$ and L is the density scale length. The point $z = 0$ now corresponds to plasma resonance $\omega = \omega_p$. The overdense region of the plasma corresponds to $z < 0$, and the upper hybrid cutoff is located at $z = LY^2$.

Since we assume $\omega > \Omega$, $Y < 1$ and we introduce the dimensionless quantities

$$\xi = 2|k_{\perp}|z/(1 - Y^2)^{1/2} \tag{6a}$$

and

$$\beta = |k_{\perp}|L\, Y^2/2(1 - Y^2)^{1/2} \tag{6b}$$

so that the homogeneous form of Eq. (3) becomes

$$\frac{d}{d\xi}\left(\xi\frac{d}{d\xi}\phi\right) - (\xi/4 - \beta)\phi = 0 \tag{7}$$

Using the transformation

$$\phi(\xi) = f(\xi)\exp(-\xi/2) \tag{8}$$

Eq. (7) becomes

$$\xi \frac{d^2}{d\xi^2} f + (1 - \xi) \frac{df}{d\xi} + (\beta - \tfrac{1}{2}) f = 0 \qquad (9)$$

Eq. (9) has the form of Kummer's differential equation which has two independent solutions usually denoted by $U(a,b,\xi)$ and $M(a,b,\xi)$. The two independent parameters a and b here have the values $a = \tfrac{1}{2} - \beta$ and $b = 1$. Since the Wronskian of U and M is zero for certain values of a and b we use a different combination of linearly independent solutions to construct the Green's function. In the notation of Slater[4], the solutions we use are

$$y_5 = U(a,b,\xi) \qquad (10)$$

and

$$y_7 = e^{\xi} U(b-a,b,-\xi)$$

The Wronskian of y_5 and y_7 is, for real ξ

$$W(y_5, y_7) = \xi^{-b} \exp\left[\xi + i\pi(a - b)\right] \qquad (11)$$

which is not zero for any finite value of a or b.

The Green's function is a solution to the equation (with $\sigma = 1$)

$$\frac{d}{d\xi}\left[\xi \frac{d}{d\xi} G(\xi,\xi_o)\right] - (\xi/4 - \beta) G(\xi,\xi_o) = -4\pi\sigma L \delta(\xi - \xi_o) \qquad (12)$$

where σ is the effective charge density per unit area of the source.

A solution to Eq. (12) can be constructed using linear combinations of the solutions to the homogeneous Eq. (7) on either side of the source. We denote these combinations by

$$G_{>} = c_{11} \exp(-\xi/2) y_5 + c_{12} \exp(-\xi/2) y_7 \quad ; \quad \xi > \xi_o \tag{13}$$

and

$$G_{<} = c_{21} \exp(-\xi/2) y_5 + c_{22} \exp(-\xi/2) y_7 \quad ; \quad \xi < \xi_o \tag{14}$$

The solutions $G_>$ and $G_<$ must satisfy the following conditions to form a physically acceptable Green's function:

1) They must be continuous at $\xi = \xi_o$

$$G_>(\xi_o) = G_<(\xi_o) \tag{15}$$

2) The derivative at $\xi = \xi_o$ must satisfy

$$\left(\frac{dG_>}{d\xi} - \frac{dG_<}{d\xi}\right)\bigg|_{\xi=\xi_o} = \frac{-4\pi\sigma L}{\xi_o} \tag{16}$$

3) Both $G_>$ and $G_<$ must vanish as $|\xi| \to \infty$ and,

4) $G_>$ and $G_<$ must be bounded for real ξ.

To address point 3 we consider the asymptotic behavior of the function U. For $|\xi| \to \infty$

$$U(a,b,\xi) \sim (\xi)^{-a} \tag{17}$$

Using Eq. (17) the asymptotic behavior of the components of $G_>$ and $G_<$ are

$$\exp(-\xi/2)y_5 \to \exp(-\xi/2)(\xi)^{\beta-\frac{1}{2}} \tag{18a}$$

and

$$\exp(-\xi/2)y_7 \to \exp(+\xi/2)(-\xi)^{-\beta-\frac{1}{2}} \tag{18b}$$

where we have used the values of a and b appropriate for Eq. (9). From Eq. (18) it is clear that in order for $G_>$ and $G_<$ to vanish as $|\xi| \to \infty$

$$c_{12} = c_{21} = 0 \tag{19}$$

Thus the Green's function must be constructed from

$$G_> = c_{11}\exp(-\xi/2)y_5 \quad ; \quad \xi > \xi_o \tag{20a}$$

$$G_< = c_{22}\exp(-\xi/2)y_7 \quad ; \quad \xi < \xi_o \tag{20b}$$

In considering the boundedness of the solutions (point 4) the behavior of the solutions at plasma resonance ($\xi = 0$) is crucial. For the case under consideration ($b = 1$) the behavior of the functions $U(a,1,\xi)$ for $\xi << 1$ is

$$U(a,1,\xi) \simeq -\ln \xi/\Gamma(a) \tag{21}$$

The logarithmic singularity for solutions near the origin when $\Gamma(a)$ is finite apparently indicates the buildup of charge near the resonant layer. In this case, the solutions $G_>$ and $G_<$ of Eq. (20) can not be used to construct a physically acceptable Green's function. This situation indicates that the cold plasma description is inadequate to describe the physical processes occurring near the resonance layer. Thermal terms must then be retained to adequately describe the plasma. However, Eq. (21) suggests the possibility of obtaining physically acceptable solutions when $\Gamma(a) = 0$, i.e. whenever a is zero or a negative integer. Physically this implies it is possible to obtain a purely cold mode which vanishes at infinity and is bounded at plasma resonance.

The parameter a can indeed have zero or negative integer values for certain perpendicular wavenumbers. Thus we expect bounded solutions to exist whenever $a = \frac{1}{2} - \beta = -j$ $(j = 0,1,2,\ldots)$. Using Eq. (6b) to express this condition in terms of physical variables, $k_\perp$ must satisfy

$$|k_\perp|L = (2j + 1)(1 - Y^2)^{\frac{1}{2}}/Y^2 \tag{22}$$

For these discrete or quantized values of $k_\perp$ the Kummer function $U(a,1,\xi)$ becomes a polynomial, namely,

$$U(-j,1,\xi) = (-1)^j j!\, L_j(\xi) \tag{23}$$

where $L_j(\xi)$ is the $j^{\underline{th}}$, zero order Laguere polynomial which is, of course, finite at the origin. Since the parameter β is a positive quantity the solution $G_<$ which is proportional to $U(\beta+\frac{1}{2},1,-\xi)$ is not bounded at the origin. Thus in addition to satisfying Eq. (22) we must also require, for a bounded Green's function, that the source be in the overdense region of the plasma, (i.e. $\xi_o < 0$) as illustrated in Fig. 2. Thus the solutions $G_>$ and $G_<$ vanish at infinity and are everywhere bounded for $a = -j$ and $\xi_o < 0$, in which case

$$G_> = c_{11} \exp(-\xi/2)(-1)^j j!\ L_j(\xi) \tag{24a}$$

$$G_> = c_1 \exp(-\xi/2)\ L_j(\xi) \quad ; \quad \xi > \xi_o$$

and

$$G_< = c_{22} \exp(-\xi/2) y_7 \tag{24b}$$

$$G_< = c_2 \exp(-\xi/2) U(j+1,1,-\xi) \quad ; \quad \xi < \xi_o$$

The constants c_1 and c_2 can be found using Eqs. (15) and (16). From Eq. (15) we find

$$c_2 = c_1 \exp(|\xi_o|)\ L_j(-|\xi_o|)/U(j+1,1,|\xi_o|) \tag{25}$$

The derivatives of $G_>$ and $G_<$ at $\xi = -\xi_o = |\xi_o|$ are

$$G_>' = c_1 \exp(|\xi_o|/2)\ [-\tfrac{1}{2} L_j(-|\xi_o|) + L_j'(-|\xi_o|)] \tag{26a}$$

$$G_<' = c_2 \exp(-|\xi_o|/2)[\tfrac{1}{2} U(1+j,1,|\xi_o|) + U'(j+1,1,|\xi_o|)] \tag{26b}$$

where the prime indicates a derivative with respect to ξ. Inserting Eqs. (26a) and (26b) in Eq. (16) and using Eq. (25) for c_2 together with the expression for the Wronskian [Eq. (11)] we obtain

$$c_1 = -4\pi\sigma L \exp[-|\xi_o|/2](j!)\ U\ (j+1,1,|\xi_o|) \tag{27a}$$

and

$$c_2 = -4\pi\sigma L \exp[|\xi_o|/2](j!)\, L_j(-|\xi_o|) \tag{27b}$$

Using Eqs. (27a,b) the Green's function appropriate for sources on the over-dense side at discrete values of $|k_\perp|$ is

$$G_> = -(j!)\, 4\pi\sigma L\, U(j+1,1,|\xi_o|)\, L_j(\xi) \exp[-(\xi - \xi_o)/2] \quad ; \quad \xi > \xi_o = -|\xi_o| \tag{28a}$$

$$G_< = -(j!)\, 4\pi\sigma L\, L_j(-|\xi_o|)\, U(j+1,1,-\xi) \exp[(\xi - \xi_o)/2] \quad ; \quad \xi < \xi_o < 0 \tag{28b}$$

III. Discussion

It has been demonstrated that for certain discrete values of perpendicular wavenumber, it is possible to construct a Green's function for electrostatic modes in a magnetized cold plasma with a longitudinal density gradient. The sources must be located in the overdense region of the plasma. An example of the Green's function for $j = 3$ in Eq. (22) is shown in Fig. 3. The Green's function is oscillatory in the region between plasma resonance and the upper hybrid cutoff point beyond which it decays exponentially as expected from WKB analysis. In the overdense region of the plasma the Green's function decays exponentially away from the source. Viewing the Green's function as representing physically a grid immersed in the plasma driven by a constant current source of amplitude $-i\omega\sigma$ it is clear that the plasma response is purely capacitive. The capacitive impedance of the plasma, which can be found from Eqs. (28a) or (28b), yields an effective plasma capacitance per unit area, C/A, given by

$$(C/A)^{-1} = (j!)\ 4\pi L\ L_j(-|\xi_o|)\ U(j+1,1,|\xi_o|) \tag{29}$$

This expression illustrates that the plasma response depends upon the zero order density gradient scale length and the details of the electrostatic mode structure.

At this point it is worth noting that the retention of terms resulting from changes in the parallel component of the plasma dielectric is crucial to obtaining these results. Neglecting the term in Eq. (3) proportional to $\partial_z \varepsilon_{\parallel}$ leads to erroneous results in at least two ways. First the behavior of the electrostatic modes near resonance would be found to diverge at nonquantized values of $|k_\perp|$ as z^{-1} rather than the correct $\ln(z)$ divergence. This illustrates that near resonance the plasma can not be treated as locally homogeneous. Second, the differential operator would be non-adjoint and any subsequently derived Green's function would be incorrect and more difficult to obtain than the correct one.

The Green's function presented here, which is derived from the cold plasma equations, is valid only for discrete values of $k_\perp$ at which the quantity $\Gamma(a)$ is infinite. At other values of $|k_\perp|$ the solutions to the cold plasma differential equation have a logarithmic singularity at plasma resonance. Actually, however, the amplitude of the electrostatic oscillations remains finite at resonance because the plasma has a non-zero temperature. Temperature dependent effects lead to the excitation of Bohm-Gross modes near plasma resonance. These thermal modes carry energy down the density gradient away from resonance and thus limit the amplitude. Such a mode conversion process is described mathematically by the fourth order differential equation that arises from Eq. (3) if thermal terms are retained in $\varepsilon_{\parallel}$. The investigation of the solutions to the fourth order equation is the subject of another paper[5] but for completeness, we note here that the ratio of the amplitude of the hot mode generated near resonance to the cold mode away from resonance is proportional to $1/\Gamma(a)$. Thus the amplitude of the hot mode is zero at the quantized values of $|k_\perp|$, consistent with the results presented here.

As we have indicated the source of excitation must be in the overdense region of the plasma. Even at the quantized values of $|k_\perp|$ purely cold solutions can not be constructed for the source in the underdense region because of the inherent singularity at resonance in the solutions $U(\beta+\frac{1}{2},1,-\xi)$. The problem is quite clearly not symmetric. Obtaining a suitable expression for the Green's function for $\xi_o > 0$ again requires the retention of thermal effects. The solutions to the fourth order differential equation remain bounded at the resonance because of production of hot modes through mode conversion. However, finding the Green's function in this case is much more complicated because four modes rather than two can be excited in the plasma. We are currently investigating this rather complex problem.

The suppression of mode conversion at discrete values of $|k_\perp|$ means that the plasma acts essentially as a loss-free waveguide. The wave energy is transported perpendicular to the field lines and density gradient. The long wavelength cold modes are not subject to Landau damping whereas the short wavelength Bohm-Gross modes created through mode conversion are. The experimental investigation of the waveguide-like property of the plasma can be carried out in present laboratory devices or the effect may be observable in absorption experiments conducted in the auroral ionosphere.

From the limited study reported here it is evident that fundamental differences exist between resonant absorption in a magnetized and unmagnetized plasma. In the unmagnetized plasma short wavelength Bohm-Gross modes are excited by direct coupling of external electromagnetic energy to plasma resonance. In the magnetized plasma excitation of Bohm-Gross modes can take place indirectly through mode conversion from cold modes and is suppressed for certain angles of propagation. Thus the introduction of a magnetic field in the plasma produces not just a change in the angular dependence of Bohm-Gross excitation but introduces a fundamentally different physical mechanism for the production of these modes.

Acknowledgement

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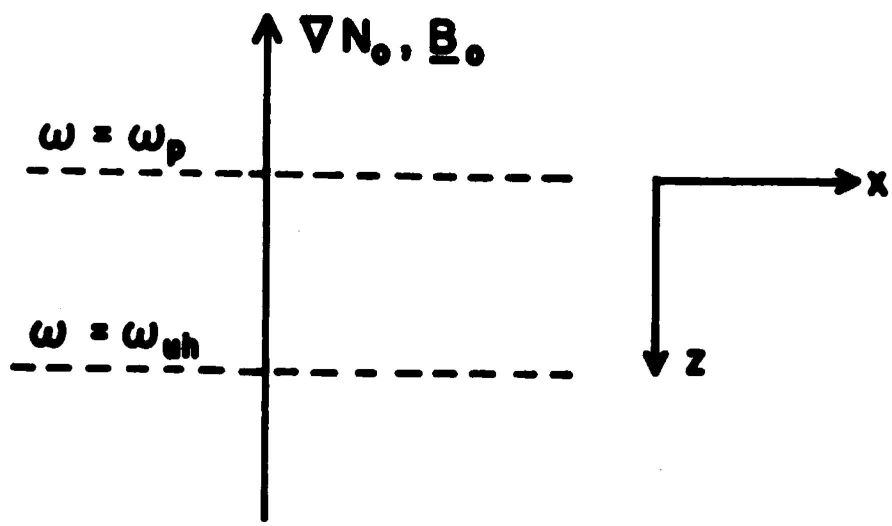
Figure Captions

Figure 1. Schematic of geometry considered in this study. The plasma density gradient ∇N_o points along the magnetic field $\underset{\sim}{B}_o$. With $\omega > \Omega$, the plasma resonance point at ω_p is near the upper hybrid point, ω_{uh}.

Figure 2. Schematic indicating the relative location of the source, plasma resonance, ω_p, and upper hybrid cut-off, ω_{uh}. To obtain bounded cold solutions the driving source must be located in the over-dense region ($\omega_p > \omega$). N_o is the plasma density.

Figure 3. Example of spatial dependence of a cold waveguide Green's function for $j = 3$. The location of the source, plasma resonance, ω_p, and upper hybrid cut-off, ω_{uh}, are indicated. The mode is finite everywhere and oscillatory between the ω_p and ω_{uh} points. ϕ_s is an arbitrary scaling factor.



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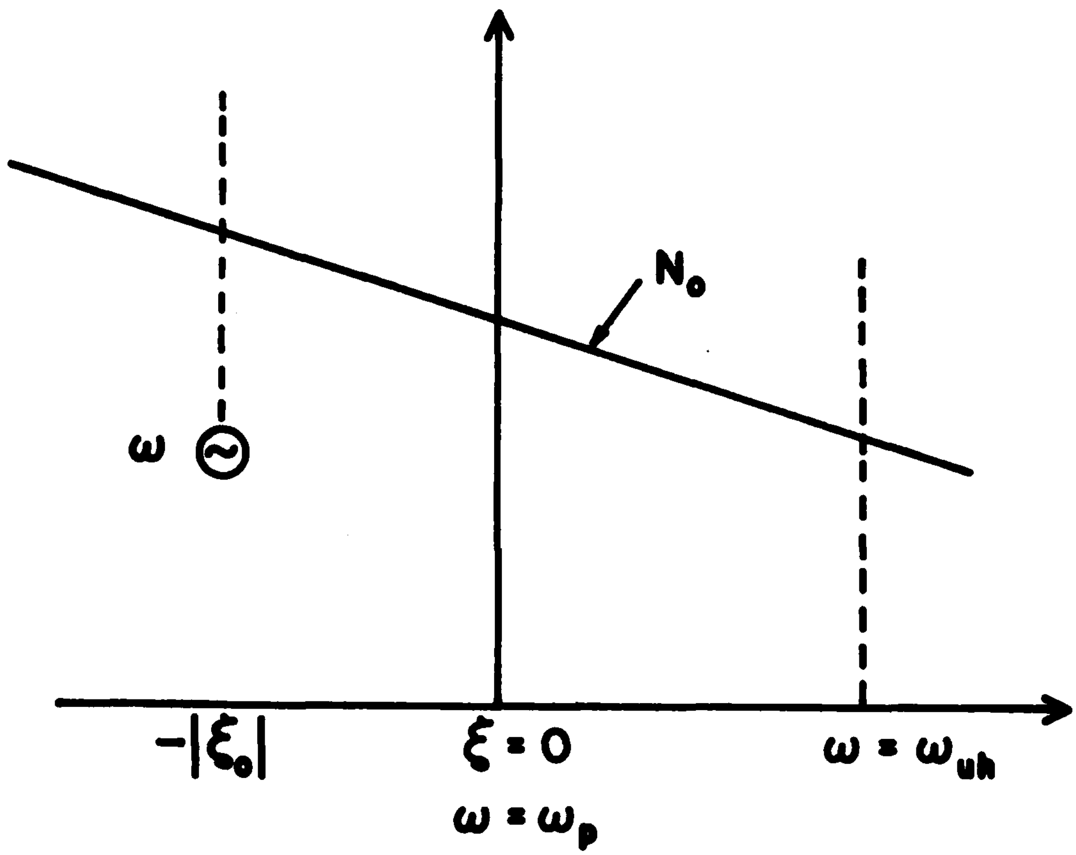
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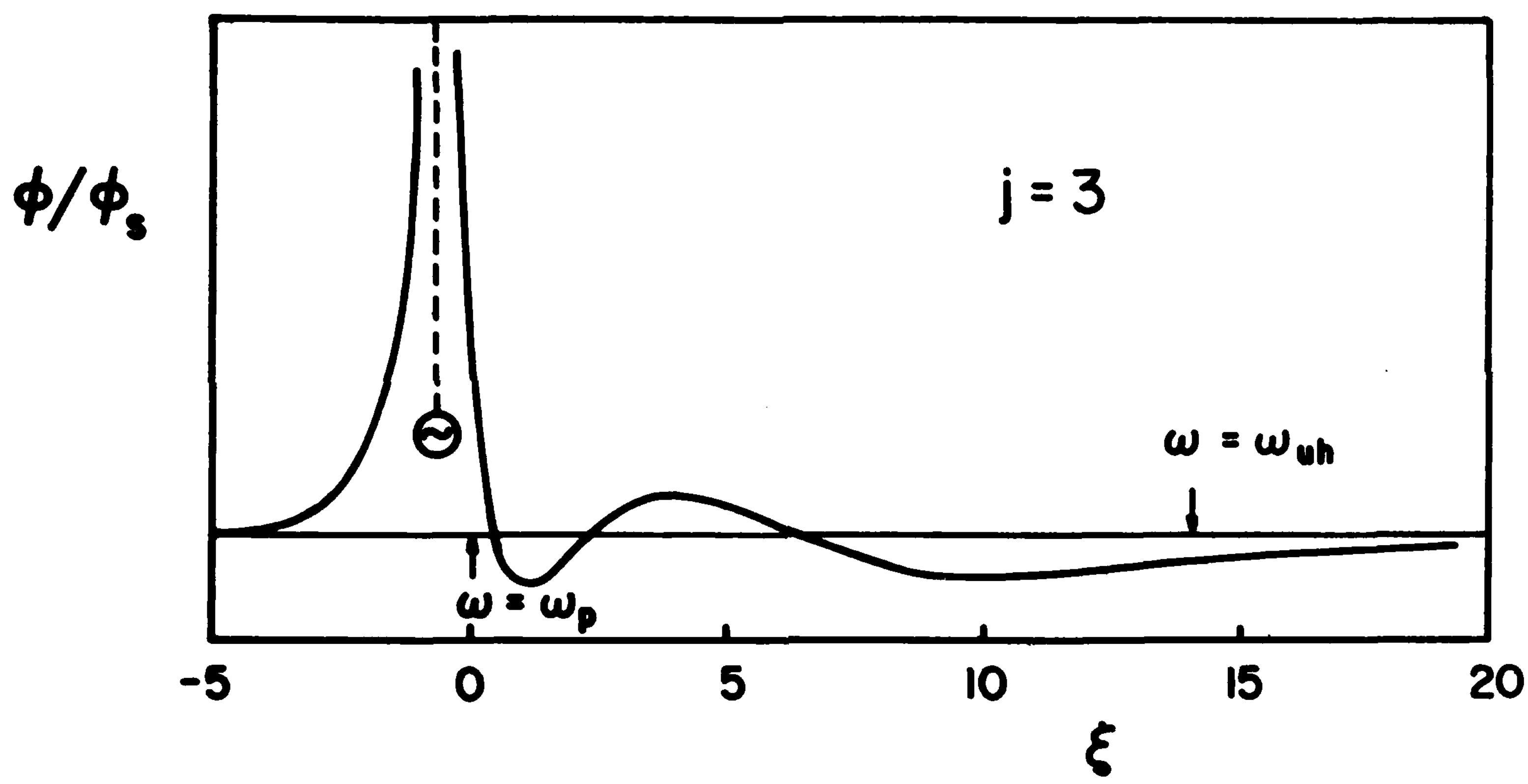

ϕ/ϕ_s
j = 3
$\omega = \omega_{uh}$
$\omega = \omega_p$
-5
0
5
10
15
20
ξ

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